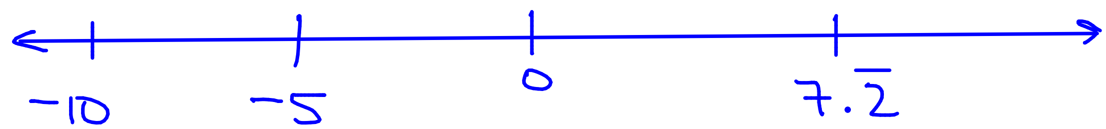
Unit 1 - Absolute Value and Radicals

1.1 Absolute Value = the distance a number is from zero on a number line.



$|-10| = 10$

$|7.\overline{2}| = 7.\overline{2}$

$|-5| = 5$

1. h) $|-3| + |2| = 5$

l) $|-4-(-2)| = 2$

$-4+2$

$|-2|$

3. arrange from least to greatest:

b) $\underbrace{-|2.\overline{5}|}_{-2.\overline{5}}$ $\underbrace{|-2.\overline{5}|}_{2.\overline{5}}$ $\underbrace{-|2.5|}_{-2.5}$ $\underbrace{|-2.5|}_{2.5}$

1 4 2 3

1.2 $\sqrt{\text{Radical}}$ Operations

$$x^n = a \qquad x = \sqrt[n]{a} = a^{\frac{1}{n}}$$

index

radicand

"n^{th} root"

If a is positive and n is even, there are two real n^{th} roots.

ex. $\sqrt{x^2} = \sqrt{81}$ $\qquad x = \pm 9$

$9^2 = 81$

$(-9)^2 = 81$

ex. $x^2 = 144$ $\qquad x = \pm 12$

$x^4 = 16$ $\qquad x = \pm 2$

If n is odd, there is only one real root.

ex. $\sqrt[3]{x^3} = \sqrt[3]{27}$ $\qquad x = 3$

$\sqrt[3]{x^3} = \sqrt[3]{-27}$ $\qquad x = -3$

$x^3 = -216$ $\qquad x = -6$

$x^3 = 8$ $\qquad x = 2$

$x^5 = -32$ $\qquad x = -2$

If n is even and a is negative.

ex. $x^2 = -16$ $\quad \emptyset$

$(?)^2 = -16$ no solutions

1. $x^5 = 32$ $\quad x = 2$

2. $x^7 = -128$ $\quad x = -2$

$(-2)^7$

3. $x^9 = 0$ $\quad x = 0$

4. $x^3 = -3$ $\quad x = \sqrt[3]{-3}$

5. $x^6 = 3$ $\quad x = \pm\sqrt[6]{3}$

6. $x^4 = -3$ $\quad$ no solution!

7. $x^4 = 16$ $\quad x = \pm 2$

Simplifying

remember

$\sqrt{20} = \sqrt{4} \times \sqrt{5} = 2\sqrt{5}$

perfect square

20: 10, 2

10: 5, 2

$\sqrt{\underbrace{2 \cdot 2} \cdot 5} = 2\sqrt{5}$

Let's say x is a real #

ex. $\sqrt[2]{x^6} \rightarrow x^{\frac{6}{2}} = |x^3|$ ← absolute value if the exponent is odd

$$\sqrt{\underbrace{x \cdot x} \cdot \underbrace{x \cdot x} \cdot \underbrace{x \cdot x}} = |x^3|$$

$$\sqrt{(x^3)^2} = |x^3|$$

ex. $\sqrt{x^4} = \sqrt{(x^2)^2} = x^2$

ex. $\sqrt{x^5} = \sqrt{x^2} \times \sqrt{x^3}$

$\downarrow$

$x\sqrt{x^3}$

$\downarrow$

$x\sqrt{x^2} \times \sqrt{x}$

$x \cdot x \times \sqrt{x}$

$x^2\sqrt{x}$

$= \sqrt{x^4} \times \sqrt{x}$

$\boxed{x^2\sqrt{x} \quad x \geq 0}$

1) $\sqrt{x^3} = \sqrt{x^2} \cdot \sqrt{x} = |x|\sqrt{x}$ $x \geq 0$

2) $\sqrt{x^4} = x^2$

3) $\sqrt{x^5} = \sqrt{x^4 \cdot x} = x^2\sqrt{x}$ $x \geq 0$

4) $\sqrt{x^6} = |x^3|$

5) $\sqrt{x^7} = \sqrt{x^6 \cdot x} = |x^3|\sqrt{x}$ $x \geq 0$

6) $\sqrt{x^8} = x^4$

7) $\sqrt{x^9} = \sqrt{x^8 \cdot x} = x^4\sqrt{x}$ $x \geq 0$

8) $\sqrt{x^{45}}$

$\sqrt{x^{44} \cdot x}$

$x^{22}\sqrt{x}$ $x \geq 0$

Simplify. All variables are real #'s.

p.10

4. a) $\sqrt{9x^2} = 3|x|$

c) $\sqrt{x^6y^4} = |x^3|y^2$

e) $\sqrt{x^2y} = |x|\sqrt{y} \quad y \geq 0$

g) $\sqrt{x^2y^2} = |xy|$

k) $\sqrt{x^5y^2} = x^2|y|\sqrt{x} \quad x \geq 0$

m) $\sqrt{16x^6y^8} = 4|x^3|y^4$

5. j) $\sqrt[4]{x^5y^7} = |xy|\sqrt[4]{xy^3} \quad x \geq 0 \quad y \geq 0$

$\sqrt[4]{\underline{x^4} \cdot x \cdot \underline{y^4} \cdot y^3}$

p. 11

6. i) $\sqrt{x^2+4x+4} = \sqrt{(x+2)(x+2)} = \sqrt{(x+2)^2} = |x+2|$

l) $\sqrt[3]{(x-1)(x^2-2x+1)} = \sqrt[3]{(x-1)^3} = x-1$

p) $\sqrt[4]{\dfrac{81}{16x^{12}}} = \dfrac{3}{2|x^3|}$

m) $\sqrt[3]{\dfrac{-4}{27x^6}} = \dfrac{\sqrt[3]{-4}}{3x^2}$

$\sqrt[3]{(x^2)^3}$

p. 11 remember... $x^{m/n} = \sqrt[n]{x^m}$

$\rightarrow (\sqrt[n]{x})^m$

#7 Re-write as a single radical.

a) $\sqrt[3]{\sqrt[4]{x^1}} = ((x)^{\frac{1}{4}})^{\frac{1}{3}}$ $(\sqrt[4]{81})^3 = 27$

$= x^{\frac{1}{12}} = \sqrt[12]{x}$

e) $\sqrt[2]{\sqrt[2]{\sqrt[2]{xy^2z^3}}} = \sqrt[8]{xy^2z^3}$

1.3 Simplifying Radicals (more...)

$\sqrt[n]{ab} = \sqrt[n]{a} \times \sqrt[n]{b}$

$\sqrt[n]{\dfrac{a}{b}} = \dfrac{\sqrt[n]{a}}{\sqrt[n]{b}}$

p. 17

4
9
16
25
36
49
64
⋮

#3. Change to simplest radical form

a) $\sqrt{32} = \sqrt{16} \cdot \sqrt{2} = 4\sqrt{2}$

c) $\sqrt{75} = \sqrt{25} \cdot \sqrt{3} = 5\sqrt{3}$

8
27
64
125
⋮

e) $\sqrt[3]{16} = \sqrt[3]{8}\cdot\sqrt[3]{2} = 2\sqrt[3]{2}$

g) $3\sqrt{45} = 3\cdot\sqrt{9}\cdot\sqrt{5} = 9\sqrt{5}$

i) $\frac{4}{3}\sqrt{54} = \frac{4}{\cancel{3}}\sqrt{\cancel{9}}\cdot\sqrt{6} = 4\sqrt{6}$

k) $\frac{5}{6}\sqrt[3]{-32} = \frac{5}{6}\sqrt[3]{-8}\cdot\sqrt[3]{4} = \frac{5}{3\cancel{6}}(-\cancel{2})\sqrt[3]{4} = -\frac{5}{3}\sqrt[3]{4}$

m) $\sqrt[4]{162} = \sqrt[4]{81}\cdot\sqrt[4]{2} = 3\sqrt[4]{2}$

4. All variables are non-negative.

a) $\sqrt{x^3y^2}$ $xy\sqrt{x}$ $\sqrt{x^2}\cdot\sqrt{x}\cdot\sqrt{y^2} = xy\sqrt{x}$

c) $\sqrt{18x^6}$ $3x^3\sqrt{2}$

e) $\sqrt{\frac{5}{x^4}}$ $\frac{\sqrt{5}}{x^2}$ $x \neq 0$

g) $\sqrt[3]{40x^3}$ $2x\sqrt[3]{5}$

i) $\sqrt[3]{-64x^3y^6}$ $-4xy^2$ $\rightarrow$ $\sqrt[3]{(-4)^3x^3(y^2)^3}$

k) $\sqrt[4]{16x^4y^8}$ $2xy^2$

m) $\sqrt[5]{-32x^4}$ $-2x^{\frac{4}{5}}$ $-2x^{\frac{4}{5}}$ ✓ $= -2\sqrt[5]{x^4}$

5. Write as an entire radical.

c) $3\sqrt[2]{4} = \sqrt{3^2 \cdot 4} = \sqrt{36}$

g) $3\sqrt[3]{4} = \sqrt{3^3 \cdot 4} = \sqrt{27 \times 4} = \sqrt{108}$

m) $2\sqrt[5]{3} = \sqrt{2^5 \cdot 3} = \sqrt{32 \times 3} = \sqrt{96}$

7. all variables are non-negative

a) $x\sqrt{y} \rightarrow \sqrt{x^2 y}$

c) $2x\sqrt[3]{5xy} \rightarrow \sqrt[3]{8x^3 \cdot 5xy} = \sqrt[3]{40x^4 y}$

g) $\dfrac{x^2\sqrt[4]{xy}}{y} \rightarrow \sqrt[4]{\dfrac{x^8 \cdot x y^1}{y^4}} = \sqrt[4]{\dfrac{x^9}{y^3}}$

j) $\dfrac{\sqrt[4]{8x}}{2x} \rightarrow \sqrt[4]{\dfrac{8x}{2^4 x^4}} = \sqrt[4]{\dfrac{\cancel{8}x}{2\cancel{16}x^4}} = \sqrt[4]{\dfrac{1}{2x^3}}$

8. Express each radical in simplest radical form. All variables are non-negative.

a) $(\sqrt{3a^2b})(\sqrt{6ab^5}) = \sqrt{18a^3b^6}$

$= \cancel{\sqrt{9}} \times \sqrt{2} \times \cancel{\sqrt{a^2}} \times \sqrt{a} \times \cancel{\sqrt{b^6}}$

$= 3ab^3\sqrt{2a}$

b) $(4x\sqrt{10xy})(3y\sqrt{2x}) = 12xy\sqrt{20x^2y}$

$= 12xy\sqrt{4}\cdot\sqrt{5}\cdot\sqrt{x^2}\cdot\sqrt{y}$

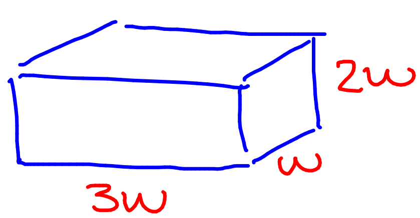
$= 12xy\cdot 2\cdot\sqrt{5}\cdot x\cdot\sqrt{y} = 24x^2y\sqrt{5y}$

c) $(2x\sqrt[3]{2y^4})(x^2\sqrt[3]{4y^2}) = 4x^3y^2$

$= 2x^3\sqrt[3]{8y^6}$

$= 2x^3(2y^2)$

11.



$V = 192\text{ cm}^3$

height is twice width $h = 2w$

length is three times width $l = 3w$

Find dimensions.

$h = 2w$

$l = 3w$

$3w(2w)w = 192$

$6w^3 = 192$

$w^3 = 32$

$w = \sqrt[3]{32}$

$w = 2\sqrt[3]{4}\text{ cm}$

$h = 2w$

$h = 4\sqrt[3]{4}\text{ cm}$

$l = 6\sqrt[3]{4}\text{ cm}$!!